HISTORY OF MATHEMATICS

Soon after language develops, it is safe to assume that humans begin counting - and that fingers and thumbs provide nature's **abacus**. The decimal system is no accident. Ten has been the basis of most counting systems in history.

When any sort of record is needed, notches in a stick or a stone are the natural solution. In the earliest surviving traces of a counting system, numbers are built up with a repeated sign for each group of 10 followed by another repeated sign for 1.

Arithmetic cannot easily develop until an efficient numerical system is in place. This is a late arrival in the story of mathematics, requiring both the concept of **place value** and the idea of **zero**.

As a result, the early history of mathematics is that of geometry and algebra. At their elementary levels the two are mirror images of each other. A number expressed as two squared can also be described as the area of a square with 2 as the length of each side. Equally 2 cubed is the volume of a cube with 2 as the length of each dimension.

Babylon and Egypt: from 1750 BC

The first surviving examples of geometrical and algebraic calculations derive from Babylon and Egypt in about 1750 BC.

Of the two Babylon is far more advanced, with quite complex algebraic problems featuring on cuneiform tablets. A typical Babylonian maths question will be expressed in geometrical terms, but the nature of its solution is essentially algebraic (see a **Babylonian maths question**). Since the numerical system is unwieldy, with a base of 60, calculation depends largely on tables (sums already worked out, with the answer given for future use), and many such tables survive on the tablets.

Egyptian mathematics is less sophisticated than that of Babylon; but an entire papyrus on the subject survives. Known as the Rhind **papyrus**, it was copied from earlier sources by the scribe Ahmes in about 1550 BC. It contains brainteasers such as problem 24: - What is the size of the heap if the heap and one seventh of the heap amount to 19?

The papyrus does introduce one essential element of algebra, in the use of a standard **algebraic symbol** - in this case *h* or *aha*, meaning 'quantity' - for an unknown number.

Pythagoras: 6th century BC

Ancient mathematics has reached the modern world largely through the work of Greeks in the classical period, building on the Babylonian tradition. A leading figure among the early Greek mathematicians is Pythagoras.

In about 529 BC Pythagoras moves from Greece to a Greek colony at Crotona, in the heel of Italy. There he establishes a philosophical sect based on the belief that numbers are the underlying and unchangeable truth of the universe. He and his followers soon make precisely the sort of discoveries to reinforce this numerical faith.

The Pythagoreans can show, for example, that musical notes vary in accordance with the length of a vibrating string; whatever length of string a lute player starts with, if it is doubled the note always falls by exactly an octave (still the basis of the scale in music today).

The followers of Pythagoras are also able to prove that whatever the shape of a triangle, its three angles always add up to the sum of two right angles (180 degrees).
most famous equation in classical mathematics is known still as the Pythagorean theorem: in any right-angle triangle the square of the longest side (the hypotenuse) is equal to the sum of the squares of the two other sides. It is unlikely that the proof of this goes back to Pythagoras himself. But the theorem is typical of the achievements of Greek mathematicians, with their primary interest in geometry.
This interest reaches its peak in the work compiled by Euclid in about 300 BC.

Euclid and Archimedes: 3rd century BC
Euclid teaches in Alexandria during the reign of Ptolemy. No details of his life are known, but his brilliance as a teacher is demonstrated in the *Elements*, his thirteen books of geometrical theorems. Many of the theorems derive from Euclid's predecessors (in particular **Eudoxus**), but Euclid presents them with a clarity which ensures the success of his work. It becomes Europe's standard textbook in geometry, retaining that position until the 19th century.

Archimedes is a student at Alexandria, possibly within the lifetime of Euclid. He returns to his native Syracuse, in Sicily, where he far exceeds the teacher in the originality of his geometrical researches.

The fame of **Archimedes** in history and legend derives largely from his practical inventions and discoveries, but he himself regards these as trivial compared to his work in pure geometry. He is most proud of his calculations of surface area and of volume in spheres and cylinders. He leaves the wish that his tomb be marked by a device of a sphere within a cylinder.

A selection of titles from his surviving treatises suggests well his range of interests: *On the Sphere and the Cylinder*; *On Conoids and Spheroids*; *On Spirals*; *The Quadrature of the Parabola*; or, closer to one of his practical discoveries, *On Floating Bodies*.

The circumference of the earth: calculated c. 220 BC

Eratosthenes, the librarian of the **museum at Alexandria**, has more on his mind than just looking after the scrolls. He is making a map of the stars (he will eventually catalogue nearly 700), and he is busy with his search for prime numbers; he does this by an infinitely laborious process now known as the **Sieve of Eratosthenes**.

But his most significant project is working out the circumference of the earth.

Eratosthenes hears that in noon at midsummer the sun shines straight down a well at Aswan, in the south of Egypt. He finds that on the same day of the year in Alexandria it casts a shadow 7.2 degrees from the vertical. If he can calculate the distance between Aswan and Alexandria, he will know the circumference of the earth (360 degrees instead of 7.2 degrees, or 50 times greater).

He discovers that camels take 50 days to make the journey from Aswan, and he measures an average day's walk by this fairly predictable beast of burden. It gives him a figure of about 46,000 km for the circumference of the earth. This is, amazingly, only 15% out (40,000 km is closer to the truth).

Algebra: from the 2nd century AD

The tradition of Babylonian algebra is revived by the Greeks in Alexandria, where Diophantus writes a treatise called *Arithmetica* in about AD 200; he uses a special sign for minus, and adopts the letter *s* for the unknown quantity. Greek algebra in its turn spreads to India, China and Japan. But it achieves its widest influence through the Arabic transmission of Greek culture.

In this the most significant event is a book written in Baghdad in about AD 825 by al-Khwarizmi. Its title is *Kitab al jabr w'al-muqabala* ('Book of Restoration and Reduction'). The success of this work in Europe provides, from part of the title (*al jabr*), the word 'algebra'.

The most important Renaissance work on algebra, written by Girolamo Cardano and published in 1545, expresses in its title the status of the art; it is called *Ars Magna*, the 'great art' as opposed to the lesser art of arithmetic. But there are still no standard symbols.

These emerge during the next century. Both plus (+) and minus (-) derive from abbreviations used in Latin manuscripts. The square root sign √ is perhaps a version of *r* for *radix* ('root' in Latin). The equal sign (=) is attributed to an English author, Robert Record, in a book of 1556. In the 17th century **Descartes** introduces the use of *x*, *y* and *z* for unknown quantities, and the convention for writing squared and cubed numbers.